

Lecture 19. Methods for solving radiative-transfer equation.

Part 2. Effects of the surface reflection on the atmospheric radiation field.

Objectives:

1. Surface reflection.
2. Inclusion of the surface reflection into the radiative transfer.

Required reading:

L02: 6.3.5

1. Surface reflection

Surfaces can modify the atmospheric radiation field by

- a) reflecting a portion of the incident radiation back into the atmosphere;
- b) emitting the thermal radiation (see Lecture 3, Kirchhoff's law);
- c) absorbing a portion of incident radiation;
- d) transmitting some incident radiation.

Two extreme types of the surface reflection:

specular reflectance and **diffuse reflectance**.

Specular reflectance is the reflectance from a perfectly smooth surface (e.g., a mirror):

Angle of incidence = Angle of reflectance

- Reflection is generally **specular** when the "roughness" of the surface is smaller than the wavelength used. In the solar spectrum (0.4 to 2 μm), reflection is therefore specular on smooth surfaces such as polished metal, still water or mirrors.

NOTE: While incoming solar light is unpolarized, reflected waves are generally polarized and Fresnel's laws can be used to determine polarization.

- Practically all real surfaces are not smooth and the surface reflection depends on the incident angle and the angle of reflection. Reflectance from such surfaces is referred to as **diffuse reflectance**.

Bi-directional reflectance distribution function (BRDF), $\rho(\mu, \varphi, -\mu', \varphi')$ is introduced to characterize the angular dependence in the surface reflection and defined as the ratio of the reflected intensity to the energy flux in the incident beam:

$$\rho(\mu, \varphi, -\mu', \varphi') = \frac{\pi dI^{\uparrow}(\tau^*, \mu, \varphi)}{I^{\downarrow}(\tau^*, -\mu', \varphi')\mu' d\Omega'} \quad [19.1]$$

NOTE: Each type of surfaces has a specific spectral **BRDF**. This plays a central role in the remote sensing of planetary surfaces.

Reciprocity law: $\rho(\mu, \varphi, -\mu', \varphi') = \rho(-\mu', \varphi', \mu, \varphi)$

Upwelling radiance is an integral over BRDF and downwelling radiance

$$I_r^{\uparrow}(\tau^*, \mu, \varphi) = \frac{1}{\pi} \int_0^{2\pi} \int_0^1 \rho(\mu, \varphi, -\mu', \varphi') I^{\downarrow}(\tau^*, -\mu', \varphi') \mu' d\mu' d\varphi' \quad [19.2]$$

Surface albedo is defined as the ratio of the surface upwelling to downwelling flux:

$$r_{sur} = \frac{F^{\uparrow}}{F^{\downarrow}} = \frac{1}{\pi} \frac{\int_0^{2\pi} \int_0^1 \int_0^{2\pi} \int_0^1 \rho(\mu, \phi, -\mu', \phi') I^{\downarrow}(-\mu', \phi') \mu' d\mu' d\phi' \mu d\mu d\phi}{\int_0^{2\pi} \int_0^1 I^{\downarrow}(-\mu', \phi') \mu' d\mu' d\phi'} \quad [19.3]$$

NOTE: Eq.[19.3] is similar to Eq.[18.4] , except that the latter is written for the collimated (direct) incident field.

Note: in general the albedo *depends on the incident radiance field*.

For collimated incidence the albedo is

$$R_s(\mu_0, \phi_0) = \int_0^{2\pi} \int_0^1 \rho(\mu, \phi, -\mu_0, \phi_0) \mu d\mu d\phi$$

The *spherical albedo* or *diffuse albedo* is integrated over all collimated incident directions

$$\bar{R}_s = 2 \int_0^1 R_s(\mu_0) \mu_0 d\mu_0$$

Types of reflection

- A surface called the **Lambert surface** if it obeys **the Lambert's Law**.

Lambert's Law of diffuse reflection: the diffusely reflected light is isotropic and unpolarized (i.e., natural light) independently of the state of polarization and the angle of the incidence light.

- **For the Lambert surface**, BRDF is independent on the directions of incident and observed light beam.

$$\rho(\mu, \varphi, -\mu', \varphi') = \rho_L \quad [19.3]$$

- **For the Lambert surface**, from Eq.[19.2], we have

$$I_r^\uparrow(\tau^*, \mu, \varphi) = \frac{\rho_L}{\pi} F^\downarrow \quad [19.4]$$

$$\text{and from Eq.[19.3], we have} \quad I_{sur}' = \rho_L \quad [19.5]$$

A **specular** surface reflects like a mirror

$$\rho(\mu, \phi, \mu', \phi') = \rho(\mu) \delta(\mu + \mu') \delta(\phi - \phi')$$

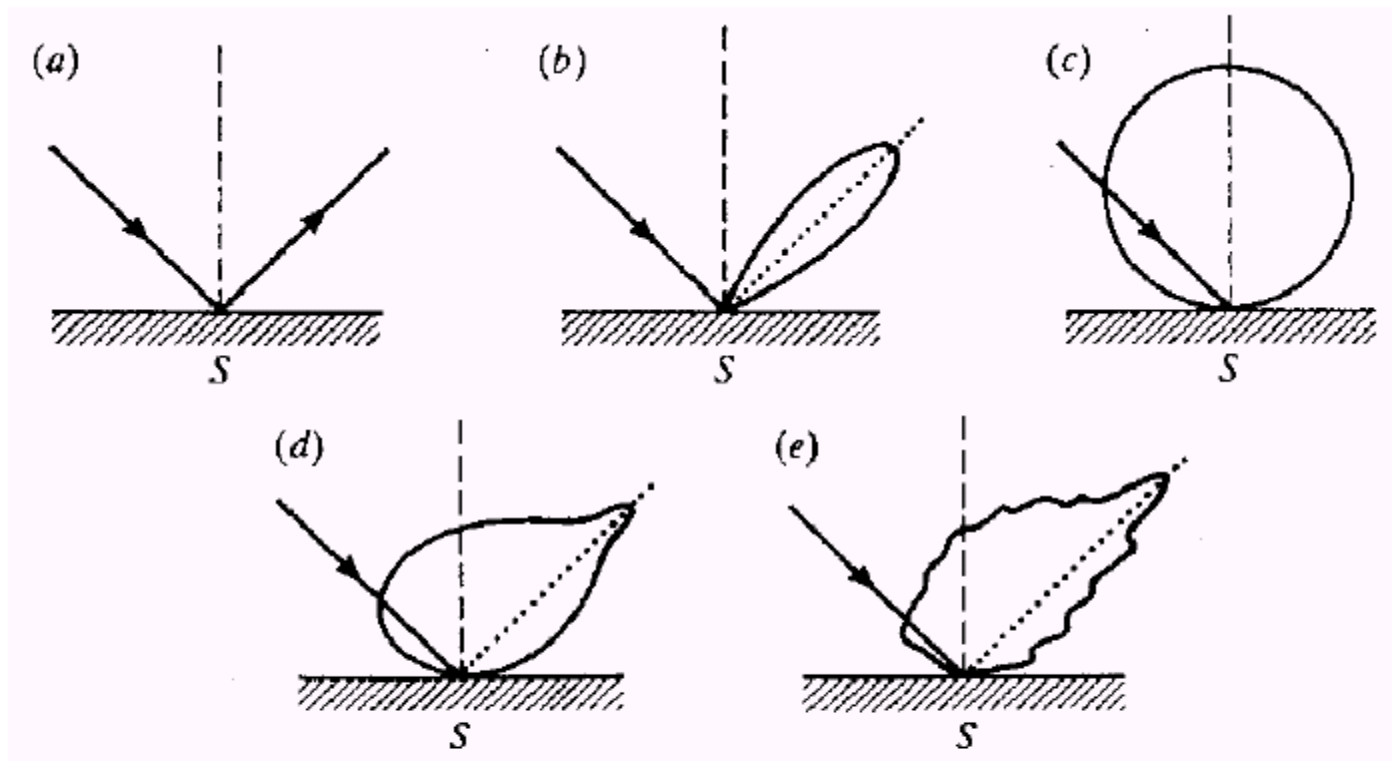


Figure 19.1 Schematic illustration of different types of surface scattering. The lobes are polar diagrams of the scattered radiation: (a) specular, (b) quasi-specular, (c) Lambertian, (d) quasi-Lambertian, (e) complex (from Rees, 1990).

Fresnel Reflection

Fresnel reflection is specular reflection by a smooth dielectric surface.

Fresnel *amplitude* reflection coefficients for each polarization ($r_{\parallel,V}$ and $r_{\perp,H}$) depend on angle θ and index of refraction m (see week 9 notes).

Intensity reflectance is $R = (|r_{\parallel}|^2 + |r_{\perp}|^2)/2$.

Reflection at nadir ($\theta = 0$): $R = \left| \frac{m-1}{m+1} \right|^2$.

Completely polarized reflection when $r_{\parallel,V} = 0$: $\cos \theta_B = \frac{1}{\sqrt{m^2+1}}$

θ_B is called Brewster's or the polarizing angle.

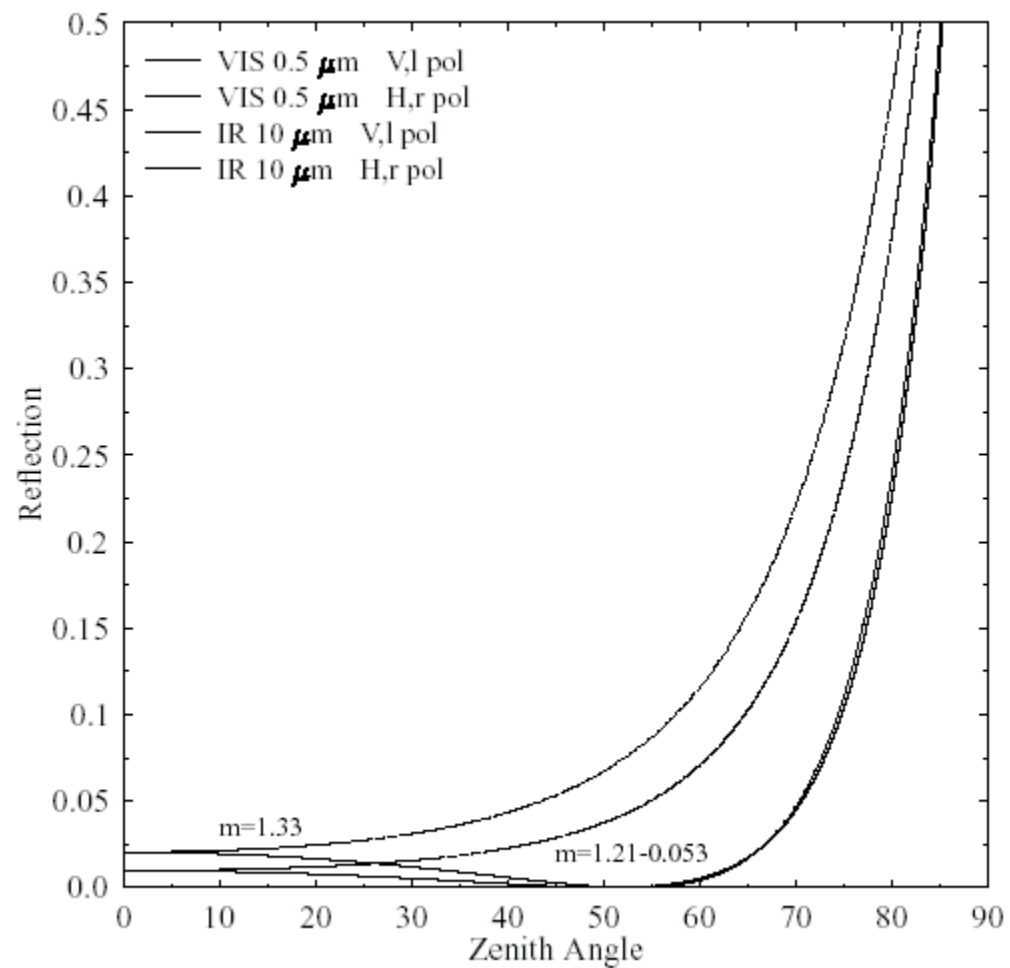
As θ_i increases horizontally polarized reflection increases monotonically; vertically polarized reflection decreases until Brewster angle, then increases. Reflection increases sharply at oblique incident angles.

➤ Ocean reflection

Ocean reflection depends on the ocean surface conditions (i.e., waves). It is often modeled by Cox and Munk (1954) model, in which wave slope distribution is Gaussian with variance proportional to wind speed. So Fresnel reflection is integrated over the orientation of facets.

NOTE: Actual visible reflection from the ocean is usually higher due to whitecaps and scattering from particles in the ocean.

Fresnel Reflection from Water



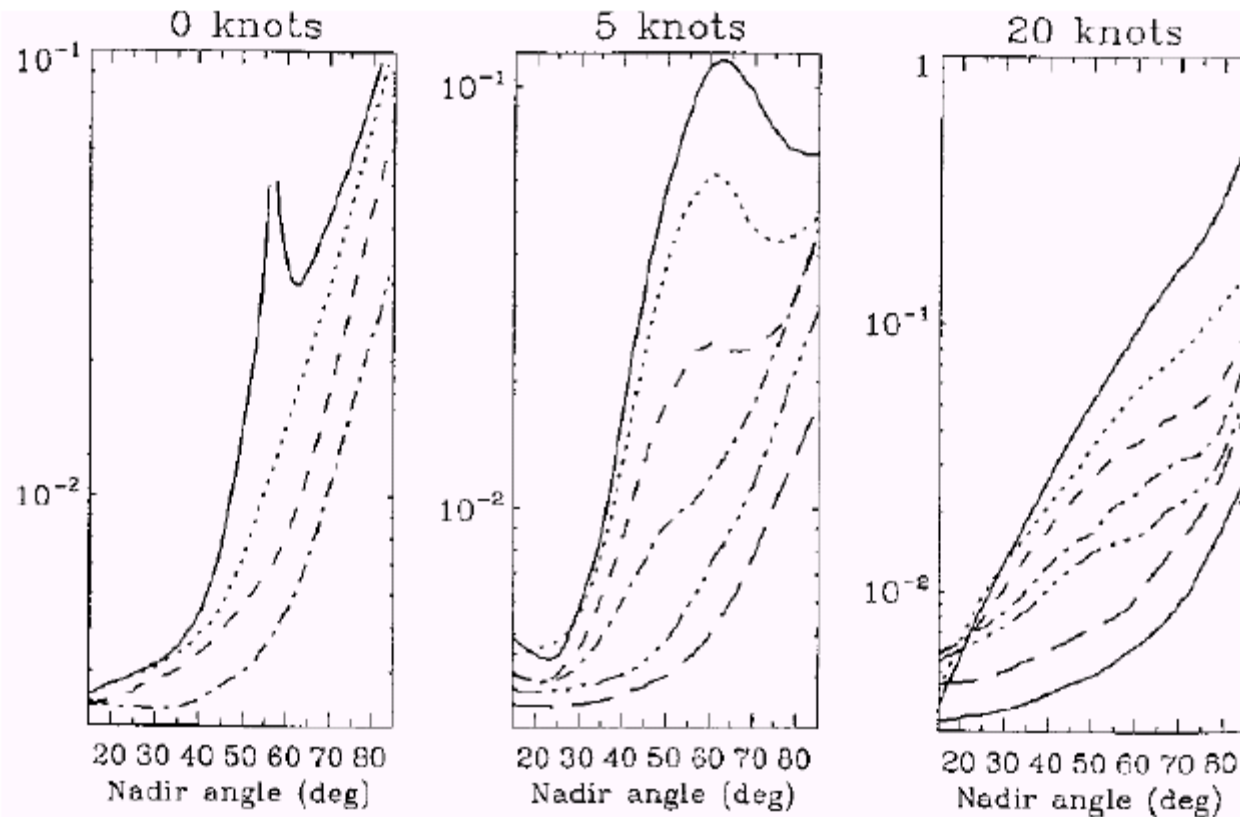


Figure 19.2 Upward intensity above the ocean surface for $\theta_0=57^\circ$ at $\lambda = 0.46 \mu\text{m}$, calculated using a Monte Carlo method and the Cox Munk model. The three plots apply to three different assumed wind speeds. Each curve is for a different azimuthal plane, where $\phi=0$ being forward direction on the plane of incidence (from Thomas and Stamnes, 1999).

➤ **Reflection from soils and vegetation**

A notable feature of the BRDF for natural surfaces is the **hot spot** - a peak in reflectance for direct backscatter ($\Theta=180^\circ$).

Hot spot are caused by:

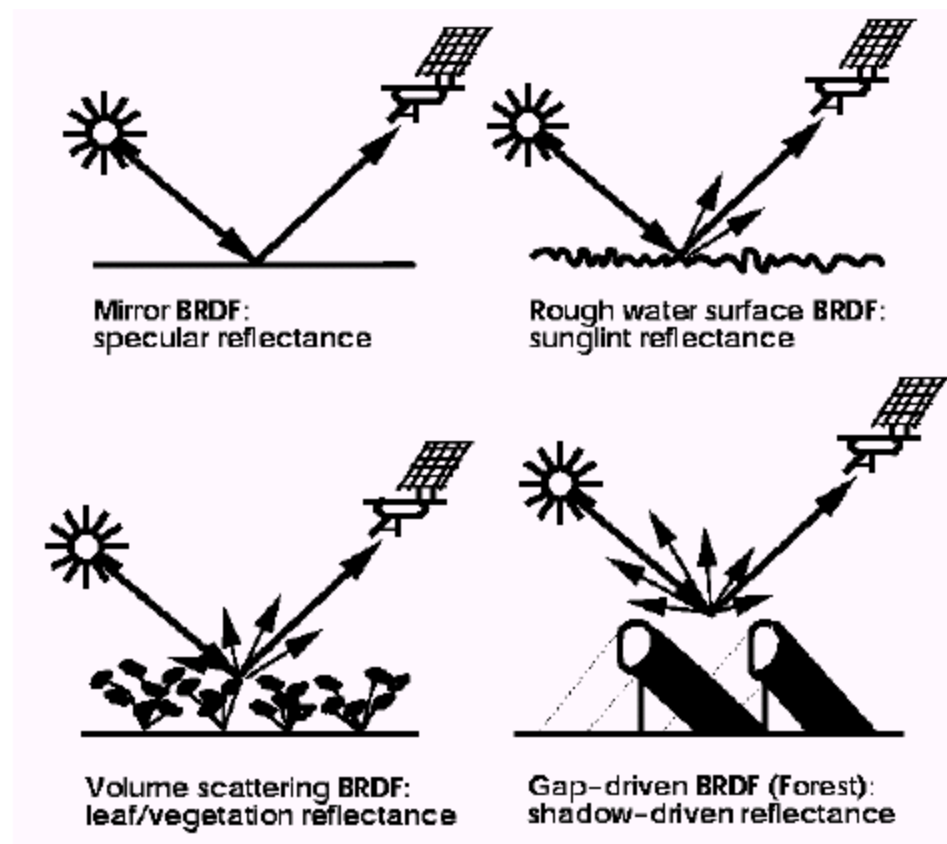
- 1) the lack of observed shadows and
- 2) specular reflection from oriented leaves

➤ **In general, the surface reflection is a function of the wavelength.**

Examples of the surface albedo at about 550 nm: fresh snow/ice =0.8-0.9; desert=0.3, soils=0.1-0.25; ocean=0.05

Spectral dependence:

- 1) vegetation has a sharp increase in reflectance around $0.7 \mu\text{m}$ (dark in visible, bright in near IR, due to chlorophyll).
- 2) Soils have a more gradual increase in albedo from VIS to NIR.
- 3) Snow has high albedo in VIS, decreases in NIR (less albedo in absorption bands and for larger grain size).



Basic reasons for land surface reflection anisotropy: specular scattering such as sunglint, also observed where forward-scattering leaves or soil elements are present; radiative transfer-type volumetric scattering by finite scatterers (leaves of plant canopies) that are uniformly distributed, potentially nonuniformly inclined and themselves have anisotropic reflectance; and geometric-optical surface scattering, which is given by shadow-casting and mutual obscuration of three-dimensional surface elements, for example of trees in a sparse forest or brushland, or of clods on a plowed field or of rock-strewn deserts. In natural systems, all types of scattering are likely to occur simultaneously.

Examples of the spectral surface reflectance.

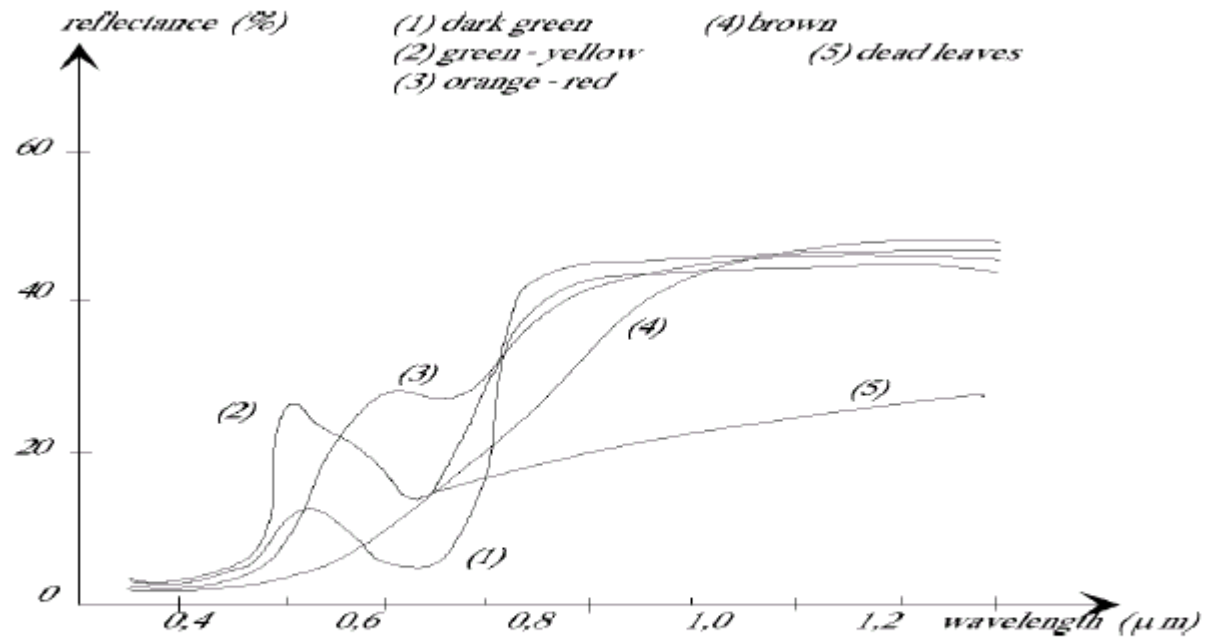


Figure 19.3 Evolution of the spectral reflectance of beech leaves during senescence (from Kniplinh, 1969).

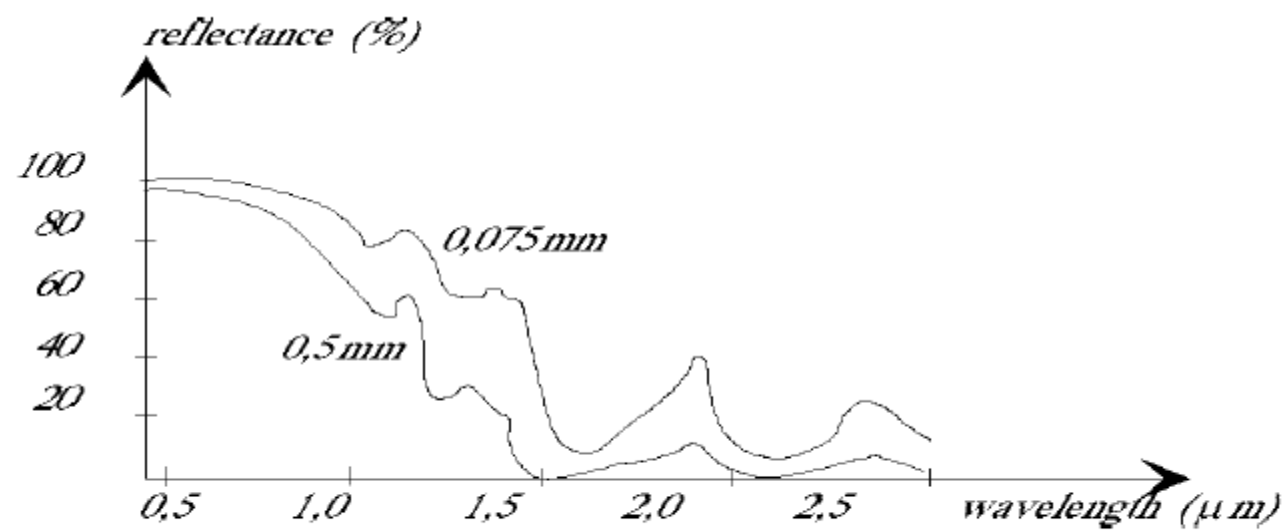


Figure 19.4 Reflectance of snow composed of crystals of $d=0.075\text{ mm}$ and $d=0.5\text{ mm}$.

Empirical Reflection Models

Empirical models of surfaces often used to fit observed reflection data.

Minnaert's formula:

$$\rho_M(\mu, \mu_0) = \rho_0 \mu^{k-1} \mu_0^{k-1}$$

A more flexible model from Rahman, Pinty, Verstraete is used for vegetation

$$\rho_{RPV}(\theta_1, \phi_1; \theta_2, \phi_2) = \rho_0 \frac{\cos^{k-1} \theta_1 \cos^{k-1} \theta_2}{(\cos \theta_1 + \cos \theta_2)^{1-k}} F(g) [1 + R(G)]$$

$$F(\Theta) = (1 - g^2) / [1 + g^2 - 2g \cos(\Theta)]^{3/2}$$

$$R(G) = \frac{1 - \rho_0}{1 + G} \quad G = \sqrt{\tan^2 \theta_1 + \tan^2 \theta_2 - 2 \tan \theta_1 \tan \theta_2 \cos(\phi_1 - \phi_2)}$$

which depends on ρ_0, k, g . $F(\Theta)$ controls forward scattering, and $1 + R(G)$ models the hot spot.

2. Inclusion of surface reflection into the radiative-transfer equation.

Let's include the contribution from the Lambert surface.

$$\textbf{Lambert surface: } I^{\uparrow}(\tau^*, \mu, \varphi) = I_{sur} = const \quad [19.6]$$

Generalizing the definitions for the reflection and transmission functions (i.e., Eqs.[18.1—[18.2]), we may express the reflected diffuse intensity $I_r^{\uparrow}(0, \mu, \varphi)$ and transmitted diffuse intensity $I_t^{\downarrow}(\tau^*, -\mu, \varphi)$ as

$$I_r^{\uparrow}(0, \mu, \varphi) = \frac{1}{\pi} \int_0^{2\pi} \int_0^1 R(\mu, \varphi, \mu', \varphi') I_{inc}(-\mu', \varphi') \mu' d\mu' d\varphi' \quad [19.7]$$

$$I_t^{\downarrow}(\tau^*, -\mu, \varphi) = \frac{1}{\pi} \int_0^{2\pi} \int_0^1 T(\mu, \varphi, \mu', \varphi') I_{inc}(-\mu', \varphi') \mu' d\mu' d\varphi' \quad [19.8]$$

The reflected intensity at the top of the layer including the surface reflection may be written as

$$I^*(0, \mu, \varphi) = I^\uparrow(0, \mu, \varphi) + \frac{1}{\pi} \int_0^{2\pi} \int_0^1 T(\mu, \varphi, \mu', \varphi') I_{sur} \mu' d\mu' d\varphi' + I_{sur} \exp(-\tau^* / \mu) \quad [19.9]$$

NOTE: The second term on the right-hand side gives the contribution from the surface reflected intensity which is diffusely transmitted to the top of the layer, whereas the third term gives the contribution from the surface reflected intensity which is the directly transmitted.

We can re-write Eq.[19.9] as

$$I^*(0, \mu, \varphi) = \mu_0 F_0 R(\mu, \varphi, \mu_0, \varphi_0) + I_{sur} \gamma(\mu) \quad [19.10]$$

where $\gamma(\mu) = \exp(-\tau^* / \mu) + t(\mu)$

Now, let's consider the diffuse transmitted intensity. Isotropic intensity I_{sur} , propagating upward in the layer after being scattered by the Lambertian surface, can be partially reflected back to the surface and, hence, contribute to the downward intensity in the additional amount

$$I_{add}^{\downarrow}(-\mu) = \frac{1}{\pi} \int_0^{2\pi} \int_0^1 R(\mu, \varphi, \mu', \varphi') I_{sur} \mu' d\mu' d\varphi' = I_{sur} r(\mu)$$

Thus, the transmitted intensity including the surface contribution is

$$I^*(\tau^*, -\mu, \varphi) = I^{\downarrow}(\tau^*, -\mu, \varphi) + I_{sur} r(\mu) = \mu_0 F_0 T(\mu, \varphi, \mu_0, \varphi_0) + I_{sur} r(\mu) \quad [19.11]$$

Both Eqs.[19.10] and [19.11] have I_{sur} . Thus, we need to find I_{sur} .

$$\pi I_{sur} = (\text{Surface albedo}) \times (\text{Downward flux})$$

The downward flux has three components:

(1) Transmitted direct flux = $\mu_0 F_0 \exp(-\tau^* / \mu_0)$

(2) Transmitted diffuse flux=

$$\int_0^{2\pi} \int_0^1 I^\downarrow(\tau^*, -\mu, \varphi) \mu d\mu d\varphi = \int_0^{2\pi} \int_0^1 \frac{\mu_0 F_0}{\pi} T(\mu, \varphi, \mu_0, \varphi_0) \mu d\mu d\varphi = \mu_0 F_0 t(\mu_0)$$

(3) Fraction of I_{sur} reflected by the atmosphere back to the surface =

$$\int_0^{2\pi} \int_0^1 I_{atm}^\downarrow(-\mu) \mu d\mu d\varphi = \pi I_{sur} \bar{r}$$

Therefore

$$\pi I_{sur} = r_{sur} (\mu_0 F_0 \exp(-\tau^* / \mu_0) + \mu_0 F_0 t(\mu_0) + \pi I_{sur} \bar{r})$$

and rearranging term, we have

$$I_{sur} = \frac{r_{sur}}{1 - r_{sur} \bar{r}} \frac{\mu_0 F_0}{\pi} \gamma(\mu_0)$$

Therefore, the diffuse reflected and transmitted intensities, accounting for the surface contribution are

$$I^*(0, \mu, \varphi) = I^\uparrow(0, \mu, \varphi) + \frac{r_{sur}}{1 - r_{sur}\bar{r}} \frac{\mu_0 F_0}{\pi} \gamma(\mu_0) \gamma(\mu) \quad [19.12a]$$

$$I^*(\tau^*, -\mu, \varphi) = I^\downarrow(\tau^*, -\mu, \varphi) + \frac{r_{sur}}{1 - r_{sur}\bar{r}} \frac{\mu_0 F_0}{\pi} \gamma(\mu_0) r(\mu) \quad [19.12b]$$

Integrating Eq.[19.12a, b] over the solid angle, we find diffuse fluxes

$$F^*(0) = F^\uparrow(0) + \frac{r_{sur}}{1 - r_{sur}\bar{r}} \mu_0 F_0 \gamma(\mu_0) \bar{\gamma} \quad [19.13a]$$

$$F^*(\tau^*) = F^\downarrow(\tau^*) + \frac{r_{sur}}{1 - r_{sur}\bar{r}} \mu_0 F_0 \gamma(\mu_0) \bar{r} \quad [19.13a]$$

where $\bar{\gamma} = \bar{t} + 2 \int_0^1 \exp(-\tau^*/\mu_0) \mu_0 d\mu_0$

NOTE: \bar{t} and \bar{r} were defined in Lecture 18 (see Eq.[18.8] and [18.9]).

NOTE: For non-Lambert surface, the inclusion of the surface reflection is a complex boundary problem.